# TALLER 12

(1) Compruebe que el vector $X$ es una solución del sistema dado

(a)
$$X' = \begin{pmatrix} -1 & 1/4 \\ 1 & -1 \end{pmatrix} X \qquad X = \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-\frac{3t}{2}}$$

$$X' = \begin{pmatrix} -1 & 1/4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -e^{-3t/2} \\ 2e^{-3t/2} \end{pmatrix} \qquad X = \begin{pmatrix} -e^{-3t/2} \\ 2e^{-3t/2} \end{pmatrix}$$

$$X' = \begin{pmatrix} \frac{3}{2} e^{-3t/2} \\ -3e^{-3t/2} \end{pmatrix}$$

$$X' = \begin{pmatrix} e^{-3t/2} + \frac{1}{4}\, 2e^{-3t/2} \\ -e^{-3t/2} - 2^{-3t/2} \end{pmatrix}$$

$$X' = \begin{pmatrix} \frac{3}{2} e^{-3t/2} \\ -3e^{-3t/2} \end{pmatrix} \checkmark$$

(b)
$$X' = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & -1 \end{pmatrix} X \qquad X = \begin{pmatrix} \operatorname{sen} t \\ -\frac{1}{2}\operatorname{sen} t - \frac{1}{2}\cos t \\ -\operatorname{sen}(t) + \cos(t) \end{pmatrix}$$

$$X' = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & -1 \end{pmatrix} X = \begin{vmatrix} \cancel{\operatorname{sen}(t)} \; \cancel{-\operatorname{sen}(t)} + \cos(t) \\ \operatorname{sen}(t) - \frac{1}{2}\operatorname{sen}(t) - \frac{1}{2}\cos(t) \\ -2\operatorname{sen}(t) + \operatorname{sen}(t) - \cos(t) \end{vmatrix}$$

$$X' = \begin{pmatrix} \cos(t) \\ -\frac{1}{2}\cos(t) + \frac{1}{2}\operatorname{sen}(t) \\ -\cos(t) - \operatorname{sen}(t) \end{pmatrix}$$

$$\begin{pmatrix} \cos(t) \\ \frac{1}{2}\operatorname{sen}(t) - \frac{1}{2}\cos(t) \\ -\operatorname{sen}(t) - \cos(t) \end{pmatrix}$$

② Los vectores dados son soluciones de un sistema $X' = AX$. Determine si los vectores propios forman un conjunto fundamental de soluciones en $(-\infty, \infty)$

ⓐ $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$ $\quad X_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-6t}$

Utilizamos el wronskiano

$$\begin{pmatrix} e^{-2t} & e^{-6t} \\ e^{-2t} & -e^{-6t} \end{pmatrix} = -e^{-8t} - e^{-8t}$$

$$= -2e^{-8t}$$

↳ $W \neq 0$ son soluciones LI y forman un CFS

ⓑ $X_1 = \begin{pmatrix} 1 \\ 6 \\ -13 \end{pmatrix}$ $\quad X_2 = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} e^{-4t}$ $\quad X_3 = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} e^{3t}$

$$\begin{vmatrix} 1 & e^{-4t} & 2e^{3t} \\ 6 & -2e^{-4t} & 3e^{3t} \\ -13 & -e^{-4t} & -2e^{3t} \end{vmatrix} = \begin{vmatrix} -2e^{-4t} & 3e^{3t} \\ -e^{-4t} & -2e^{3t} \end{vmatrix} - 6\begin{vmatrix} e^{-4t} & 2e^{3t} \\ -e^{-4t} & -2e^{3t} \end{vmatrix}$$

$$-13\begin{vmatrix} e^{-4t} & 2e^{3t} \\ -2e^{-4t} & 3e^{3t} \end{vmatrix}$$

$$-4e^{-t} + 3e^{t} - 6\left[\underbrace{-2e^{-t} + 2e^{-t}}_{\to 0}\right] - 13\left[3e^{-t} + 4e^{-t}\right]$$

$$-e^{-t} - 13\left[7e^{-t}\right]$$

$W \neq 0$ son solucione LI y forman CFS

(3) Compruebe que el vector $X_p$ es una solucion particular del sistema dado

(a)

$$X' = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} X + \begin{pmatrix} -5 \\ 2 \end{pmatrix} \quad ; \quad X_p = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$X' = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad X'_p = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2+3 \\ 1-3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \checkmark$$

(b)

$$X' = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} X - \begin{pmatrix} 1 \\ 7 \end{pmatrix} e^{t} \qquad X_p = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{t}$$

$$X_p = \begin{pmatrix} e^{t} \\ e^{t} \end{pmatrix} + \begin{pmatrix} te^{t} \\ -te^{t} \end{pmatrix}$$

$$X_p = e^{t} \begin{pmatrix} 1+t \\ 1-t \end{pmatrix}$$

4 Encuentre la solucion general des SIS. ECN

(a) $X' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} X$

$\begin{pmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{pmatrix} = 0$

$= (2-\lambda)(-2-\lambda)+3=0$

$= -4-\cancel{2\lambda}+\cancel{2\lambda}+\lambda^2+3=0$

$= \lambda^2-1=0$

$\lambda^2=1$

$\lambda=1 \quad \lambda=-1$

$\lambda=1$

$\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$V_1 - V_2 = 0$

$V_1 = V_2 \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda=-1$

$\begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$3V_1 - V_2 = 0$

$3V_1 = V_2 \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$X(t) = e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}C_1 + e^{-t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}C_2$

(b) $X' = \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix} X$

$\begin{vmatrix} -6-\lambda & 2 \\ -3 & 1-\lambda \end{vmatrix} = 0$

$(-6-\lambda)(1-\lambda)+6=0$

$\cancel{-6}+6\lambda-\lambda+\lambda^2+\cancel{6}=0$

$\lambda^2+5\lambda=0$

$\lambda(\lambda+5)=0$

$\lambda=0$

$\begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-3V_1 + V_2 = 0$

$V_2 = 3V_1 \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\lambda = -5$

$$\begin{pmatrix} -1 & 2 \\ -3 & 6 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-V_1 + 2V_2 = 0 \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$2V_2 = V_1$

$$X_{(t)} = C_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-5t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(C) $X' = \begin{pmatrix} -1 & 3 \\ -3 & 5 \end{pmatrix} X$

$$\begin{vmatrix} -1-\lambda & 3 \\ -3 & 5-\lambda \end{vmatrix} = 0$$

$(-1-\lambda)(5-\lambda) + 9 = 0$

$-5 + \lambda - 5\lambda + \lambda^2 + 9 = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 2)^2 = 0$

$\lambda = 2$

$$\begin{pmatrix} -3 & 3 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-3V_1 + 3V_2 = 0 \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$3V_2 = 3V_1$

valor propio repetido

$$\begin{pmatrix} -3 & 3 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{matrix} 1 \\ 1 \end{matrix}$$

$-3P_1 + 3P_2 = 1 \quad \begin{pmatrix} 1 \\ 4/3 \end{pmatrix}$

$3P_2 = 1 + 3P_1$

$$X_{(t)} = e^{2t} \left[ C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} t + \begin{pmatrix} 1 \\ 4/3 \end{pmatrix} \right.\right.$$

(D) $X' = \begin{pmatrix} 4 & -5 \\ 5 & -4 \end{pmatrix} X$

$$\begin{vmatrix} 4-\lambda & -5 \\ 5 & -4-\lambda \end{vmatrix} = 0$$

$(4-\lambda)(-4-\lambda) + 25 = 0$

$-16 \cancel{-4\lambda} \cancel{+4\lambda} + \lambda^2 + 25 = 0$

$\lambda^2 + 9 = 0$

$\lambda = \pm 3i$

$\lambda = 3i$

$$\begin{pmatrix} 4-3i & -5 \\ 5 & -4-3i \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4-3i & -5 \\ 5 & -4-3i \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ \frac{4-3i}{5} \end{pmatrix} e^{(0+3i)t}$$

$$(4-3i\,v_1) - 5v_2 = 0$$

$$4-3i\,v_1 = 5v_2$$

$$\begin{pmatrix} 1 \\ \frac{4}{5} - \frac{3i}{5} \end{pmatrix} e^{3it} = \begin{pmatrix} 1 \\ \frac{4}{5} - \frac{3i}{5} \end{pmatrix} \cos(3t) + i\,\text{sen}(3t)$$

$$\begin{pmatrix} \cos(3t) + i\,\text{sen}(3t) \\ \frac{4}{3}\cos(3t) + \frac{4}{3} i\,\text{sen}(3t) - \frac{3i}{5}\cos(3t) + \frac{3}{5}\text{sen}(3t) \end{pmatrix}$$

$$\begin{pmatrix} \cos(3t) \\ \frac{4}{3}\cos(3t) + \frac{3}{5}\text{sen}(3t) \end{pmatrix} + i \begin{pmatrix} \text{sen}(3t) \\ \frac{4}{3}\text{sen}(3t) - \frac{3}{5}\cos(3t) \end{pmatrix}$$

$$X(t) = C_1 \begin{pmatrix} \cos(3t) \\ \frac{4}{3}\cos(3t) + \frac{3}{5}\text{sen}(3t) \end{pmatrix} + C_2 \begin{pmatrix} \text{sen}(3t) \\ \frac{4}{3}\text{sen}(3t - \frac{3}{5}\cos(3t) \end{pmatrix}$$

5

(a) Se sabe que $r = -\frac{1}{2} + i$ es un valor propio de una matriz real $A$ $2\times 2$ si $\xi \begin{pmatrix}1\\ i\end{pmatrix}$ es un vector propio de $A$ asociado al valor propio $r$ entonces encuentre la solución del sistema $X' = AX$

$$\begin{pmatrix}1\\ i\end{pmatrix} e^{-t/2} e^{it} = e^{-t/2}\left[\begin{pmatrix}1\\ i\end{pmatrix} e^{it}\right] = e^{-t/2}\left[\begin{pmatrix}1\\ i\end{pmatrix} \cos(t) + i\,\mathrm{sen}(t)\right]$$

$$e^{-t/2}\begin{bmatrix}\cos(t) + i\,\mathrm{sen}(t)\\ i\cos(t) - \mathrm{sen}(t)\end{bmatrix} = e^{-t/2}\left[\begin{pmatrix}\cos(t)\\ -\mathrm{sen}(t)\end{pmatrix} + i\begin{pmatrix}\mathrm{sen}(t)\\ \cos(t)\end{pmatrix}\right]$$

$$X(t) = e^{-t/2}\left[c_1\begin{pmatrix}\cos(t)\\ -\mathrm{sen}(t)\end{pmatrix} + c_2\begin{pmatrix}\mathrm{sen}(t)\\ \cos(t)\end{pmatrix}\right]$$

(b) Se sabe que $r = 5 + 2i$ es un valor propio de una matriz real $A$, $2\times 2$ si $\xi \begin{pmatrix}1\\ 1-2i\end{pmatrix}$ es un vector propio de $A$ asociado al valor propio $r$ entonces encuentre la solución general del sistema $X' = AX$

$$\begin{pmatrix}1\\ 1-2i\end{pmatrix} e^{5t}\cdot e^{2it} = e^{5t}\left[\begin{pmatrix}1\\ 1-2i\end{pmatrix} e^{2it}\right] = e^{5t}\left[\begin{pmatrix}1\\ 1-2i\end{pmatrix} \cos(2t) + i\,\mathrm{sen}(2t)\right]$$

$$e^{5t}\left[\begin{pmatrix}\cos(2t) + i\,\mathrm{sen}(2t)\\ \cos(2t) + i\,\mathrm{sen}(2t) - 2i\cos(2t) + 2\,\mathrm{sen}(2t)\end{pmatrix}\right]$$

$$e^{5t}\left[\begin{pmatrix}\cos(2t)\\ \cos(2t) + 2\,\mathrm{sen}(2t)\end{pmatrix} + i\begin{pmatrix}\mathrm{sen}(2t)\\ \mathrm{sen}(2t) - 2\cos(2t)\end{pmatrix}\right]$$

$$X(t) = e^{5t}\left[c_1\begin{pmatrix}\cos(2t)\\ \cos(2t) + 2\,\mathrm{sen}(2t)\end{pmatrix} + c_2\begin{pmatrix}\mathrm{sen}(2t)\\ \mathrm{sen}(2t) - 2\cos(2t)\end{pmatrix}\right]$$

(6) Se sabe que $r=-3$ es un valor propio repetido de la matriz

$$A = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}$$ con multiplicidad geometrica 1 y $\xi = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ es un vector propio de A asociado al valor propio $r=-3$ Encuentre la solución del PVI

$$X' = AX \quad X(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$4P_1 - 4P_2 = 1$$

$$4P_1 = 1 + 4P_2 \qquad \begin{pmatrix} 5/4 \\ 1 \end{pmatrix}$$

$$P_1 = \frac{1}{4} + \frac{4}{4}P_2$$

$$X(t) = e^{-3t}\left[C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}t + \begin{pmatrix} 5/4 \\ 1 \end{pmatrix}\right]\right]$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2\begin{pmatrix} 5/4 \\ 1 \end{pmatrix}$$

$$3 = C_1 + \frac{5C_2}{4}$$

$$2 = C_1 + C_2$$

$$1 = \frac{C_2}{4}$$

$$C_2 = 4$$

$$2 = C_1 + 4$$

$$C_1 = -2$$

$$\vec{X}(t) = e^{-3t}\left[\begin{pmatrix}-2\\-2\end{pmatrix} + \begin{pmatrix}4\\4\end{pmatrix}t + \begin{pmatrix}5\\4\end{pmatrix}\right]$$

(7) Un sistema masa resorte de dos masas y 3 resortes satisface las ecuaciones diferenciales

$$U_1'' = -2U_1 + U_2 \qquad U_2'' = U_1 - 2U_2 \qquad (1)$$

Si $X_1 = U_1$ $\quad X_3 = U_2$ $\quad$ y $\vec{X} = \begin{pmatrix}X_1\\X_2\\X_3\\X_4\end{pmatrix}$

$X_2 = U_1'$ $\quad X_4 = U_2'$

$$\vec{X} = \begin{pmatrix}U_1\\U_1'\\U_2\\U_2'\end{pmatrix} \qquad \vec{X}' = \begin{pmatrix}U_1'\\U_1''\\U_2'\\U_2''\end{pmatrix} \qquad \vec{X}' = \begin{pmatrix}X_2\\-2U_1+U_2\\X_4\\U_1-2U_2\end{pmatrix}$$

$$\vec{X}' = \begin{pmatrix}X_2\\-2X_1+X_3\\X_4\\X_1-2X_3\end{pmatrix} \qquad \vec{X}' = \begin{pmatrix}0&1&0&0\\-2&0&1&0\\0&0&0&1\\1&0&-2&0\end{pmatrix}\vec{X}$$

8) Considere el sistema lineal homogeneo

$$X' = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix} X$$

Un vector propio de la Matriz de coeficientes y una solucion $X(t)$ del sistema correspondiente a este vector propio estan dados por

$$\begin{pmatrix} \sim \\ \sim \\ \sim \end{pmatrix} e^{4t}$$

$$\begin{pmatrix} 4-4 & 6 & 6 \\ 1 & 3-4 & 2 \\ -1 & -4 & -3-4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 6 & 6 \\ 1 & -1 & 2 \\ -1 & -4 & -7 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$6V_2 + 6V_3 = 0$$

$$V_2 = -V_3$$

$$V_1 - V_2 + 2V_3 = 0$$

$$V_1 + V_3 + 2V_3$$

$$V_1 = -3V_3$$

$$\boxed{\begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix}}$$

(9) Sea A una matriz 2X2 con valores propios $r_1 = -1$ y $r_2 = -2$ tal que $\xi^{(1)} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\xi^{(2)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Encuentre la solucion general

$$X(t) = c_1 e^{-t} \begin{pmatrix} -1 \\ 2 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

(10) Encuentre dos soluciones linealmente independientes del sistema homogeneo en $(-\infty, \infty)$

$$X' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} X$$

$$\begin{pmatrix} 2-\lambda & -5 \\ 1 & -2-\lambda \end{pmatrix}$$

$$(2-\lambda)(-2-\lambda) + 5 = 0$$

$$-4 \cancel{-2\lambda} \cancel{+2\lambda} + \lambda^2 + 5 = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

$\lambda = i$

$$= \begin{pmatrix} 2-i & -5 \\ 1 & -2-i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(2-i)v_1 - 5v_2 = 0$$

$$v_1 (2-i) v_1 = 5 v_2$$

$$v_2 = \frac{(2-i) v_1}{5}$$

$$v_1 = \begin{pmatrix} 5 \\ 2-i \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 2-i \end{pmatrix} e^0 e^{it} \Big| \begin{pmatrix} 5 \\ 2-i \end{pmatrix} e^{it} \Big| \begin{pmatrix} 5 \\ 2-i \end{pmatrix} \cos(t) + i \operatorname{sen}(t)$$

$$5\cos(t) + 5i\operatorname{sen}(t)$$
$$2\cos(t) + 2i\operatorname{sen}(t) - i\cos(t) + \operatorname{sen}(t)$$

$$\begin{pmatrix} 5\cos(t) \\ 2\cos(t) + \operatorname{sen}(t) \end{pmatrix} + i \begin{pmatrix} 5\operatorname{sen}(t) \\ 2\operatorname{sen}(t) - \cos(t) \end{pmatrix}$$

$$X(t) = C_1 \begin{pmatrix} 5\cos(t) \\ 2\cos(t)+\operatorname{sen}(t) \end{pmatrix} + C_2 \begin{pmatrix} 5\operatorname{sen}(t) \\ 2\operatorname{sen}(t) - \cos(t) \end{pmatrix}$$

La soluciones ya son LI

Ademas no hay ningun escalar que las haga que sean LD

(11) Encuentre la Solucion general del siguiente sistema de ecuaciones

$$X' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} X$$

$$\begin{vmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{vmatrix} = 0$$

$$3-\lambda \begin{vmatrix} -\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix} - 2 \begin{vmatrix} 2 & 4 \\ 2 & 3-\lambda \end{vmatrix} + 4 \begin{vmatrix} 2 & 4 \\ -\lambda & 2 \end{vmatrix}$$

$$3-\lambda\,[-\lambda(3-\lambda)-4] - 2[2(3-\lambda)-8] + 4[4+4\lambda] = 0$$

$$3-\lambda\,[-3\lambda+\lambda^2-4] - 2[6-2\lambda-8] + 16+16\lambda = 0$$

$$\cancel{-9\lambda} + \cancel{3\lambda^2} - 12 + 3\lambda^2 - \cancel{\lambda^3} + \cancel{4\lambda} - 12 + \cancel{4\lambda} + \cancel{16} + \cancel{16} + \cancel{16\lambda} = 0$$

$$-\lambda^3 + 6\lambda^2 + 15\lambda + 8$$

$\pm 1 \pm 2 \pm 4 \pm 8$

| | | | | |
|---|---|---|---|---|
| $-1$ | $+6$ | $+15$ | $+8$ | $-1$ |
| | $1$ | $-7$ | $-8$ | |
| $-1$ | $7$ | $8$ | $0$ | |

$(\lambda+1)$

$(-\lambda^2+7x+8)(\lambda+1)$

$(\lambda-8)(\lambda+1)(-\lambda-1)$

$\lambda=8$

$\lambda=-1$

$\lambda=-1$

(12)

(a) Halle la solución general del siguiente sistema de ecuaciones

$$X' = \begin{pmatrix} -3 & 5/2 \\ -5/2 & 2 \end{pmatrix} X$$

$$\begin{pmatrix} -3-\lambda & 5/2 \\ -5/2 & 2-\lambda \end{pmatrix} = 0$$

$$(-3-\lambda)(2-\lambda) + \frac{25}{4} = 0$$

$$-6 + 3\lambda - 2\lambda + \lambda^2 + \frac{25}{4} = 0$$

$$\lambda^2 + \lambda + \frac{25}{4} - \frac{64}{4}$$

$$\lambda^2 + \lambda + \frac{1}{4} = 0$$

$$\left(\lambda + \frac{1}{2}\right)^2 = 0$$

$$\lambda = -\frac{1}{2}$$

$$\begin{pmatrix} -3+\frac{1}{2} & 5/2 \\ -5/2 & 2+\frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} -5/2 & 5/2 \\ -5/2 & 5/2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-\frac{5V_1}{2} + \frac{5V_2}{2} = 0$$

$$-5V_1 + 5V_2 = 0$$

$$-V_1 + V_2 = 0$$

$$V_2 = V_1 \qquad \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} -5/2 & 5/2 \\ -5/2 & 5/2 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$-\frac{5P_1}{2} + \frac{5P_2}{2} = 5$$

$$-5P_1 + 5P_2 = 10$$

$$-P_1 + P_2 = 2$$

$$P_2 = 2 + P_1 \qquad \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$X(t) = e^{-t/2}\left[c_1 \begin{pmatrix} 5 \\ 5 \end{pmatrix} + c_2\left[t\begin{pmatrix} 5 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}\right]\right]$$

(b) Halle la solucion del problema de valor inicial conformado por el Sistema del literal (a) y la condicion

$$X(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 5 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$-1 = 5c_1 + c_2$$
$$1 = 5c_1 + 3c_2$$
$$-2 = 0 - 2c_2$$
$$\boxed{c_2 = 1}$$

$$\boxed{c_1 \ -\frac{2}{5}}$$

$$X(t) = e^{-t/t} \left[ \begin{pmatrix} -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 5t+1 \\ 5t+3 \end{pmatrix} \right]$$

(c) Halle la solucion del problema de valor inicial conformado por el sistema del literal (a) y la condicion

$$X(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \end{pmatrix} = c_1 \begin{pmatrix} 5 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$c_2 = -3/2$$

$$2 = 5c_1 + c_2$$
$$-1 = 5c_1 + 3c_2$$

$$2 = 5c_1 - \frac{3}{2}$$

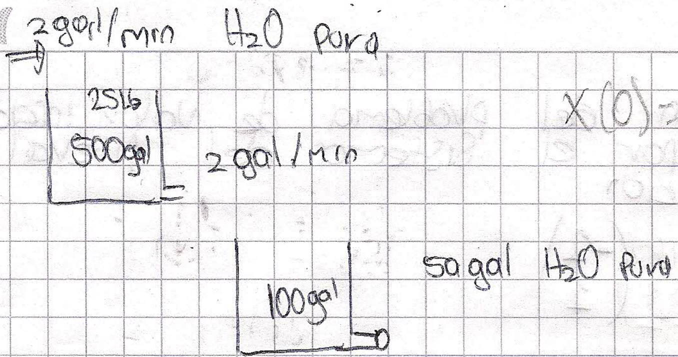
$$3 = 0 - 2c_2$$

$$\frac{22}{2} + \frac{3}{2} = 5c_1$$

$$\frac{7}{15} = c_1$$

$$\frac{7}{2} = 5c_1$$

(13)




$X(0) = 25$ lb

$x_1'(t) = C_1 V_1 - C_2 V_2$

$0 \cdot 2 - \frac{x_1}{500} 2$

$x_1'(t) = \frac{-2x_1}{500}$

$x_2'(t) = C_1 U_1 - C_2 V_2$

$= 2 \frac{x_1}{500} - 0$

$x_2'(t) = \frac{2x_1}{500}$

$X' = \begin{pmatrix} -1/250 \\ 1/250 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} x_2$

$X' = \begin{pmatrix} -1/250 & 0 \\ 1/250 & 0 \end{pmatrix} X$

$\begin{vmatrix} \frac{-1}{250} - \lambda & 0 \\ 1/250 & -\lambda \end{vmatrix} = 0$

$\left(-\frac{1}{250} - \lambda\right)(-\lambda) = 0$

$\frac{\lambda}{250} + \lambda^2 = 0$

$\lambda\left(\lambda + \frac{1}{250}\right) = 0$

$\lambda = 0$

$\lambda = -\frac{1}{250}$

$\lambda = 0$

$\frac{V_1}{250} + 0 = 0$

$V_1 = 0 \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\lambda = -1/250$

$\frac{1}{250} V_1 + \frac{V_2}{250} = 0$

$V_1 + V_2 = 0$

$V_1 = -V_2 \qquad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(14) Encuentre la solucion general del sistema de ecuaciones

$x'(t) = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} x(t) + \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t$

$\begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$

$(1-\lambda)^2 - 4 = 0$

$1 - 2\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda - 3)(\lambda + 1)$

$\lambda = 3$

$\begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-2V_1 + V_2 = 0$

$V_2 = 2V_1 \qquad \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda = -1$

$$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$2v_1 + v_2 = 0 \qquad \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$v_2 = -2v_1$$

$$X_c(t) = c_1 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\varphi = \begin{vmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{vmatrix} \qquad \det \varphi = -2e^{2t} - 2e^{2t}$$

$$= -4e^{2t}$$

$$\varphi^{-1} = \frac{1}{4e^{2t}} \begin{vmatrix} -2e^{-t} & -e^{-t} \\ -2e^{3t} & e^{3t} \end{vmatrix} = \begin{pmatrix} \frac{-e^{-3t}}{2} & \frac{-e^{-3t}}{4} \\ \frac{-e^{t}}{2} & \frac{e^{t}}{4} \end{pmatrix} \begin{pmatrix} 2e \\ -e \end{pmatrix}$$

$$X_p = \varphi \int \varphi^{-1} \cdot g(t) \, dt$$

$$X_p = \varphi \int \begin{pmatrix} -e^{-2t} + \frac{e^{-2t}}{4} \\ -e^{2t} - \frac{e^{2t}}{4} \end{pmatrix}$$

$$X_p = \psi \begin{pmatrix} \frac{e^{-2t}}{2} - \frac{e^{-2t}}{8} \\ \frac{-e^{2t}}{2} - \frac{e^{2t}}{4} \end{pmatrix} \quad \frac{3e^{-2t}}{8} \quad \frac{-3\ e^{2t}}{4}$$

$$\begin{pmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{pmatrix} \begin{pmatrix} \frac{3e^{-2t}}{8} \\ \frac{-3e^{2t}}{4} \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{8}e^{t} - \frac{3}{4}e^{t} \\ \frac{6}{8}e^{t} + \frac{6}{4}e^{t} \end{pmatrix}$$

$$e^{t} \begin{pmatrix} \frac{3}{8} - \frac{3}{4} \\ \frac{6}{8} + \frac{6}{4} \end{pmatrix} = X_p = e^{t} \begin{pmatrix} -\frac{3}{8} \\ \frac{9}{4} \end{pmatrix}$$

(15) Encuentre la solucion del PVI

$$x'(t) = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x(t) + \begin{pmatrix} -\cos t \\ \operatorname{sen} t \end{pmatrix}$$

$$x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2-\lambda & -5 \\ 1 & -2-\lambda \end{pmatrix} = 0$$

$$(2-\lambda)(-2-\lambda) + 5 = 0$$

$$-4 - \cancel{2\lambda} + \cancel{2\lambda} + \lambda^2 + 5 = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

$\lambda = i$

$$\begin{pmatrix} 2-i & -5 \\ 1 & -2-i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$v_1 = v_2(2+i)$$

$$x(t) = C_1 \begin{pmatrix} 5\cos(t) \\ 2\cos(t) + \operatorname{sen}(t) \end{pmatrix}$$

$$+ C_2 \begin{pmatrix} 5\operatorname{sen}(t) \\ 2\operatorname{sen}(t) - \cos(t) \end{pmatrix}$$

$$\Psi = \begin{pmatrix} 5\cos(t) & 5\,sen(t) \\ 2\cos(t) + sen(t) & 2sen(t) - \cos(t) \end{pmatrix}$$

$$\det \Psi = \cancel{10\,sen(t)\cos(t)} - 5\cos^2(t) \cancel{- 10\,sen(t)\cos(t)} - 5sen^2(t)$$

$$\det \Psi = -5\left(\cos^2(t) + sen^2(t)\right)$$

$$\det \Psi = -5$$

$$\Psi^{-1} = -\frac{1}{5}\begin{pmatrix} 2sen(t) - \cos(t) & -5\,sen(t) \\ -2\cos(t) + sen(t) & 5\cos(t) \end{pmatrix}$$

$$X_p = \Psi \int \Psi^{-1}\, g(t)\, dt$$

$$X_p = -\frac{1}{5}\,\Psi \int \begin{pmatrix} 2sen(t) - \cos(t) & -5\,sen(t) \\ -\cos(t) + sen(t) & 5\cos(t) \end{pmatrix} \begin{pmatrix} -\cos(t) \\ sen(t) \end{pmatrix}$$

$$X_p = -\frac{1}{5}\,\Psi \int \begin{pmatrix} -2sen(t)\cos(t) + \cos^2(t) - 5\,sen^2(t) \\ + \cos^2(t) - sen(t)\cos(t) + 5\cos(t)\,sen(t) \end{pmatrix} dt$$

$$X_p = -\frac{1}{5}\,\Psi \int \begin{pmatrix} -sen(2t) + \frac{1+\cos(2t)}{2} - \frac{5}{2}\left(\frac{1-\cos(2t)}{2}\right) \\ \frac{1+\cos(2t)}{2} + 2sen(2t) \end{pmatrix} dt$$

$$X_p = -\frac{1}{5}\,\Psi \int \begin{pmatrix} -sen(2t) + \frac{1}{2} + \frac{\cos(2t)}{2} - \frac{5}{2} + \frac{5\cos(2t)}{2} \\ \frac{1}{2} + \frac{\cos(2t)}{2} + 2\,sen(2t \end{pmatrix}$$

$$X_p = -\frac{1}{5}\,\Psi \int \begin{pmatrix} -\operatorname{Sen}(2t) + 3\cos(2t) - 2 \\ \frac{1}{2} + \frac{\cos(2t)}{2} + 2\operatorname{Sen}(2t) \end{pmatrix} dt$$

$$X_p = -\frac{1}{5}\,\Psi \begin{pmatrix} \frac{\cos(2t)}{2} - \frac{3\operatorname{Sen}(2t)}{2} - 2t \\ \frac{t}{2} + \frac{\operatorname{Sen}(2t)}{4} - \cos(2t) \end{pmatrix}$$

$$X_p = -\frac{1}{5} \begin{pmatrix} 5\cos(t) & 5\operatorname{sen}(t) \\ 2\cos(t) + \operatorname{sen}(t) & 2\operatorname{sen}(t) - \cos(t) \end{pmatrix} \begin{pmatrix} \frac{\cos(2t) - 3\operatorname{sen}(2t) - 4t}{2} \\ \frac{t + 8\operatorname{sen}(2t) - 2\cos(2t)}{2} \end{pmatrix}$$

$$X_p = -\frac{1}{10}$$

16) Halle una solucion particular para el sistema dado

(a) $X'(t) = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} X + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t}$

$$\begin{vmatrix} 0-\lambda & 2 \\ -1 & 3-\lambda \end{vmatrix} = 0$$

$$-\lambda(3-\lambda) + 2 = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda-2)(\lambda-1)$$

$\underline{\lambda = 2}$

$$\begin{pmatrix} -2 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-V_1 + V_2 = 0 \qquad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$V_2 = V_1$$

$\lambda = 1$

$$\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-V_1 + 2V_2 = 0 \qquad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$2V_2 = V_1$$

$$X_c(t) = C_1 e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Psi = \begin{pmatrix} e^{2t} & 2e^{t} \\ e^{2t} & e^{t} \end{pmatrix}$$

$$\det \Psi = e^{3t} - 2e^{3t} = -e^{3t}$$

$$\Psi^{-1} = \frac{-1}{e^{3t}}\begin{pmatrix} e^{t} & -2e^{t} \\ -e^{2t} & e^{2t} \end{pmatrix}$$

$$\Psi^{-1} = \begin{pmatrix} -e^{-2t} & 2e^{-2t} \\ e^{-t} & -e^{-t} \end{pmatrix}$$

$$\Psi^{-1}\cdot g(t) = \begin{pmatrix} -e^{-2t} & 2e^{-2t} \\ e^{-t} & -e^{-t} \end{pmatrix}\begin{pmatrix} e^{t} \\ -e^{t} \end{pmatrix}$$

$$\Psi^{-1}\cdot g = \begin{pmatrix} -e^{-t} & -2e^{-t} \\ 1 & 1 \end{pmatrix}$$

$$X_p = \Psi \int \begin{pmatrix} -3e^{-t} \\ 2 \end{pmatrix} dt$$

$$X_p = \Psi \begin{pmatrix} +3e^{-t} \\ 2t \end{pmatrix}$$

$$X_p = \begin{pmatrix} e^{2t} & 2e^{t} \\ e^{2t} & e^{t} \end{pmatrix}\begin{pmatrix} 3e^{-t} \\ 2t \end{pmatrix}$$

$$X_p = \begin{pmatrix} 3e^{t} + 4te^{t} \\ 3e^{t} + 2te^{t} \end{pmatrix}$$

$$\vec{X}(t) = C_1 e^{2t}\begin{pmatrix}1\\1\end{pmatrix} + C_2\, e^{t}\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}3\\3\end{pmatrix}e^{t} + \begin{pmatrix}4\\2\end{pmatrix}te^{t}$$

Hallemos $C_1$ y $C_2$ con $\vec{X}(0) = \begin{pmatrix}0\\2\end{pmatrix}$

$$\begin{pmatrix}C_1\\C_2\end{pmatrix} = \Psi^{-1}(0)\left[X(0) - \vec{X}p(0)\right]$$

$$\begin{pmatrix}C_1\\C_2\end{pmatrix} = \begin{pmatrix}-1 & 2\\1 & 1\end{pmatrix}\left[\begin{pmatrix}0\\2\end{pmatrix} - \begin{pmatrix}3\\3\end{pmatrix}\right]$$

$$\begin{pmatrix}C_1\\C_2\end{pmatrix} = \begin{pmatrix}-1 & 2\\1 & -1\end{pmatrix}\cdot\begin{pmatrix}-3\\-1\end{pmatrix}$$

$$\begin{pmatrix}C_1\\C_2\end{pmatrix} = \begin{pmatrix}3-2\\-3+1\end{pmatrix} \quad \Big| \quad \begin{pmatrix}C_1\\C_2\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$$

$$\boxed{\vec{X}(t) = e^{2t}\begin{pmatrix}1\\1\end{pmatrix} - 2\,e^{t}\begin{pmatrix}2\\1\end{pmatrix} + e^{t}\begin{pmatrix}3\\3\end{pmatrix}}$$

b) $\vec{X}'(t) = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\vec{X} + \begin{pmatrix}\sec(t)\\0\end{pmatrix}$

$\begin{vmatrix}-\lambda & -1\\1 & -\lambda\end{vmatrix} = 0$ | $\begin{pmatrix}-i & -1\\1 & -i\end{pmatrix}\begin{pmatrix}V_1\\V_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ | $\begin{pmatrix}1\\-i\end{pmatrix}e^{it}$

$\lambda^2 + 1 = 0$ | $-iV_1 - V_2 = 0$ | $\begin{pmatrix}1\\-i\end{pmatrix}\cos(t) + i\,\mathrm{sen}(t)$

$\lambda = \pm i$ | $-iV_1 = V_2$ | $\begin{pmatrix}\cos(t) + i\,\mathrm{sen}(t)\\-i\cos(t) + \mathrm{sen}(t)\end{pmatrix}$

$\begin{pmatrix}1\\-i\end{pmatrix}$

$$\begin{pmatrix}\cos(t)\\ \operatorname{sen}(t)\end{pmatrix} + i\begin{pmatrix}\operatorname{sen}(t)\\ -\cos(t)\end{pmatrix}$$

$$X_c(t) = c_1\begin{pmatrix}\cos(t)\\ \operatorname{sen}(t)\end{pmatrix} + c_2\begin{pmatrix}\operatorname{sen}(t)\\ \cos(t)\end{pmatrix}$$

$$\Psi = \begin{pmatrix}\cos(t) & \operatorname{sen}(t)\\ \operatorname{sen}(t) & -\cos(t)\end{pmatrix}$$

$$\det\Psi = -\cos^2(t) - \operatorname{sen}^2(t)$$

$$\det\Psi = -1$$

$$\Psi^{-1} = -1\begin{pmatrix}-\cos(t) & -\operatorname{sen}(t)\\ -\operatorname{sen}(t) & \cos(t)\end{pmatrix}$$

$$\Psi^{-1}\cdot g(t) = -1\begin{pmatrix}-\cos(t) & -\operatorname{sen}(t)\\ -\operatorname{sen}(t) & \cos(t)\end{pmatrix}\begin{pmatrix}\sec(t)\\ 0\end{pmatrix}$$

$$\Psi^{-1}\cdot g(t) = -1\begin{pmatrix}-1 + 0\\ -\tan(t) + 0\end{pmatrix}$$

$$\Psi^{-1}\cdot g(t) = \begin{pmatrix}1\\ \tan(t)\end{pmatrix}$$

$$X_p = \Psi\int\begin{pmatrix}1\\ \tan(t)\end{pmatrix}dt$$

$$X_p = \Psi\begin{pmatrix}t\\ \ln|\sec(t)|\end{pmatrix}$$

$$X_p =$$

$\Lambda^2(\mathbb{R}^3) = C$

$f \longrightarrow \nabla f \longrightarrow \operatorname{rot}(F) \longrightarrow \operatorname{div}(F) \longrightarrow 0$

$0 \longrightarrow \Lambda^0(\mathbb{R}^3) \longrightarrow \Lambda^1(\mathbb{R}^3) \xrightarrow{\text{im}} \Lambda^2(\mathbb{R}^3) \longrightarrow \Lambda^3(\mathbb{R}^3) \longrightarrow 0$

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